Middle School Math Key Concepts

Glossary:

Composite Number – (1) A whole number greater than 1 with more than two whole-number factors. (2) A whole number greater than 1 that is divisible by at least one positive integer other than itself or 1.

Examples: $6 = 1 \times 6$ | $20 = 1 \times 20$ | $100 = 1 \times 100$
$6 = 2 \times 3$ | $20 = 2 \times 10$ | $100 = 2 \times 50$
$20 = 4 \times 5$ | $100 = 4 \times 25$
$100 = 5 \times 20$
$100 = 10 \times 10$

Decimal – (1) A number that uses place value and a decimal point to show tenths, hundredths, thousandths and so on.
(2) Numbers that have a decimal point.
Example: 3.47
Expanded Form is $3 + 0.4 + 0.07$

Denominator – The bottom portion of a fraction.
Example: in $\frac{a}{b}$, b is the denominator

Exponent – The number that tells how many times to multiply the base by itself. It is often referred to as the power that a number is raised to.
Example: $2 \times 2 \times 2 \times 2 \times 2 = 2^5$
In this example, 2 is the base and 5 is the exponent.
It is referred to as two to the fifth power, or two to the power of five.
Note If the power is two then it is said that the number is squared.

Factor – (1) An integer that is multiplied by another integer to find a product.
Example: $6 \times 2 = 12$ the factors of 12 are 6 and 2
(2) To find the factors of a number.
Example: find the factors of 6
$6 \times 1 = 6$ and $2 \times 3 = 6$
so the factors of 6 are 1, 2, 3, and 6

Factor Tree – A diagram that shows the prime factors of a number.

Fraction – A number that names part of a whole or part of a group. It is in the form a/b or $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$. a is the numerator and b is the denominator.
Example: 1/3, or $\frac{1}{3}$

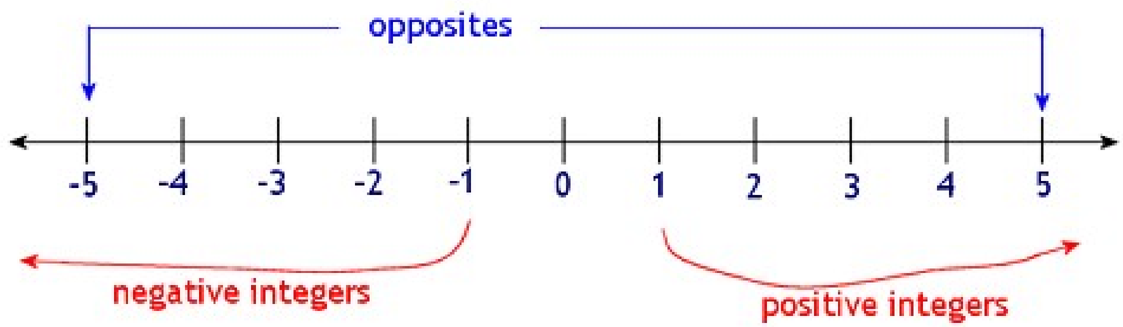
Greatest Common Factor (GCF) – The largest number that is a factor of two or more numbers.

Example: 6 is the GCF of 18 and 30

Improper Fractions – Fractions that have a numerator that is equal to or larger than the denominator.

Examples: $\frac{12}{5}, \frac{4}{4}$

Integers – The set of whole numbers and their opposites.



Irrational Numbers – Numbers that when expressed have a decimal that is neither repeating nor ending and cannot be expressed as a fraction. All numbers that are not rational.

Example: pi, $\sqrt{3}$

Least Common Denominator (LCD) – The smallest number, other than zero, that is a multiple of two or more denominators.

Example: $\frac{1}{4}$ and $\frac{1}{6}$

4 and 6 are the denominators and the LCM of 4 and 6 is 12
so the LCD of 4 and 6 is 12

now, $\frac{1}{4} = \frac{1\times 3}{4\times 3} = \frac{3}{12}$ and $\frac{1}{6} = \frac{1\times 2}{6\times 2} = \frac{2}{12}$

Least Common Multiple (LCM) – The smallest number, other than zero, that is a multiple of two or more given numbers.

Example: The LCM of 6 and 9 is 18

Mixed Numbers – Numbers that have a whole number and a fraction.

Example: $1\frac{2}{3}, 5\frac{7}{8}$

Multiple – A number that is the product of a given number and a whole number.

Example: To find the multiples of any number, multiply the number by the counting numbers 1, 2, 3, 4 and so on. The first six multiples of 3 are shown below:

$3 \times 1 = 3$, $3 \times 2 = 6$, $3 \times 3 = 9$, $3 \times 4 = 12$, $3 \times 5 = 15$, $3 \times 6 = 18$

Numerator – The top portion of a fraction.

Example: in $\frac{a}{b}$, a is the numerator

Order of Operations – The order in which operations are done; 1) do the operations in parentheses; 2) clear exponents; 3) multiply and divide from left to right; and 4) add and subtract from left to right.

Percent – The ratio of a number to 100; percent means "per hundred"

Example: 25% = 25/100

Place Value – The value of a digit as determined by its position in a number.

Prime Factorization – A number written as the product of all its prime factors.

Example: $24 = 2 \times 2 \times 2 \times 3$, or $2^3 \times 3$

Prime Number – A whole number greater than 1 that only has factors of itself and 1.

Examples: 17 has factors of 17 and 1 only
29 has factors of 29 and 1 only

Ratio – A comparison of two numbers or quantities.

Example: 3 to 5, or 3:5, or $\frac{3}{5}$

Rational Number – Any number that can be expressed as a ratio in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

Example: 0.5, 3/5, -3, 8, $3\frac{9}{10}$

Reciprocal – One of two numbers that has a product of 1. The reciprocal of x is $\frac{1}{x}$.

Examples: 2 is the reciprocal of $\frac{1}{2}$

$\frac{1}{2}$ is the reciprocal of 2

$\frac{12}{31}$ is the reciprocal of $\frac{31}{12}$

$\frac{31}{12}$ is the reciprocal of $\frac{12}{31}$

Real Numbers – The set of rational numbers and the set of irrational numbers

KEY CONCEPTS FOR MIDDLE SCHOOL MATH:

All main definitions to terms are in the glossary, so you will want to keep it handy!

I. Decimals:

A. Adding Decimals

Note When adding decimals always line up the decimal points in the numbers then add the whole numbers.

Examples: (1) 26.98 + 14.75

1st line up the decimal points

2nd add the whole numbers

$$\begin{array}{r} 26.98 \\ +\ 14.75 \\ \hline 41.73 \end{array}$$

(2) 15.234 + 8.9

1st line up the decimal points

You may add 0's to make the numbers have the same amount of places after the decimal point.

2nd add the whole numbers

$$\begin{array}{r} 15.234 \\ +\ 8.900 \\ \hline 24.134 \end{array}$$

B. Subtracting Decimals

Note When subtracting decimals always line up the decimal points in the numbers, then subtract the whole numbers.

Examples: (1) 24.879 – 14.681

1st line up the decimal points

2nd subtract the whole numbers

$$\begin{array}{r} 24.879 \\ -\ 14.681 \\ \hline 10.198 \end{array}$$

(2) 36.4 – 18.006

1st line up the decimal points

You may add 0's to make the numbers have the same amount of places after the decimal point.

2nd subtract the whole numbers

$$\begin{array}{r} 36.400 \\ -\ \underline{18.006} \\ 18.394 \end{array}$$

Assignment: Add and Subtract the Following Decimals

1. 4.5 + 2.8
2. 6 + 9.7
3. 0.234 + 0.045
4. 5.56 + 3.2
5. 8 + 0.564
6. 2.3 - 1.1
7. 0.345 - 0.289
8. 4 - 0.5
9. 3.5 - 2.8
10. 4.35 - 2.13

C. Multiplying Decimals

Multiply decimal numbers the same way as whole numbers then determine how many numbers you have to have to the right of the decimal point. To figure out how many numbers you need to place to the right of the decimal point simply add the number of digits to the right of the decimal point in both numbers.

Examples: (1) 6.079×3

1st multiply the numbers

$6079 \times 3 = 18237$

2nd determine where the decimal point goes, for this example there are three decimal places in 6.079 and none in 3 so place the decimal three places from the right in the answer.

$6.079 \times 3 = 18.237$

(2) 4.78×5.609

1st multiply the numbers

$478 \times 5609 = 2681102$

2nd determine where the decimal point goes, for this example there are two decimal places in 4.78 and three in 5.609 for a total of five decimal places, so place the decimal five places from the right in the answer.

$4.78 \times 5.609 = 26.81102$

D. Dividing Decimals

(1) By whole numbers
When dividing decimals by whole numbers, use long division and line up the decimal places.

Example: 30.24 $\div$ 14

$$14\overline{)30.24} = 2.16$$

```
     2.16
14 )30.24
  - 28
     22
    -14
      84
    - 84
       0
```

(2) By decimal numbers
When dividing decimals by decimals, you need to multiply the denominator by a power of 10 big enough to get a whole number. You then multiply the numerator by the same power of 10. Using these two numbers use long division and line up the decimal place.

Example: 1.21 $\div$ 1.1

Multiply the denominator, 1.1, and the numerator, 1.21, by 10 to 11 and 12.1. We multiply only by 10 because the numerator, 1.1, has only one place to the right of its decimal point.

$$1.1\overline{)1.21} = 11\overline{)12.1}$$

Now do long division

```
    1.1
11 )12.1
  - 11
     11
    -11
      0
```

Assignment: Multiply and Divide the following Decimals

1. 0.4×2.8
2. 8×2.5
3. $.03 \times .05$
4. 6.2×2.8
5. 12.7×0.6
6. $2.8 \div 2$
7. $0.455 \div 5$
8. $3.6 \div 1.2$
9. $4.8 \div 0.003$
10. $0.675 \div 2.5$

II. Prime and Composite Numbers:

Task: Make a chart of the numbers from 1 to 100 and state if they are composite or prime.

One way to do this is by making a chart of the first one hundred whole numbers (1-100):

1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100

Next, cross out 1, because it is not prime. Then, circle 2, because it is the smallest positive even prime. Now cross out every multiple of 2; in other words, cross out every 2nd number. Then circle 3, the next prime. Then cross out all of the multiples of 3; in other words, every third number. Some, like 6, may have already been crossed out since they may be multiples of 2. Then circle the next open number, 5. Now cross out all of the multiples of 5, or every 5th number. Continue doing this until all the numbers through 100 are either circled or crossed out. Now, if you have remembered your multiplication tables, you have just circled all the prime numbers less than 100. You don't have to stop at 100 - you can go up as far as you want to find all the prime numbers you want to find. But, as you can see, there is a lot of multiplication involved.

To find the prime factors of a number, you can use a factor tree.

Examples: (1) Find the prime factors of 28

28
□ □
2 14
□ □
2 7

Therefore, the prime factors of 28 are 2 × 2 × 7 and the prime factorization is $28 = 2 \times 2 \times 7$ or $2^2 \times 7$ (using exponents).

(2) Find the prime factors of 36.

36
□ □
3 12

3 4

2 2

Therefore, the prime factors of 36 are $2 \times 2 \times 3 \times 3$ and the prime factorization is $36 = 2 \times 2 \times 3 \times 3$ or $2^2 \times 3^2$ (using exponents).

Assignment: Prime and Composite Numbers

Identify each of the following numbers as prime or composite.

1. 14
2. 23
3. 81
4. 55

Find the prime factorization of the following numbers. Do not use exponents in your answers. If a number is prime, write prime.

5. 25
6. 24
7. 17
8. 30

III. Finding Greatest Common Factors (GCF) and Least Common Multiples (LCM):

GCF Example: Find the GCF of 16 and 24

1st find all the prime factors of the numbers
prime factorization of $16 = 2 \times 2 \times 2 \times 2$
prime factorization of $24 = 2 \times 2 \times 2 \times 3$
2nd find all the common factors in the numbers
for 16 and 24 there are three 2's in both
3rd Multiply these factors and get your GCF
$2 \times 2 \times 2 = 8$, therefore the GCF is 8

LCM Example: Find the LCM of 4 and 6

1st Find multiples of the given numbers
$4 \rightarrow$ 4, 8, 12, 16, 20, ……
$6 \rightarrow$ 6, 12, 18, 24, ….
2nd Find the first same multiple and that is the LCM
For 4 and 6 the first same multiple is 12, therefore the LCM is 12

Assignment: GCF and LCM

1. Write all the factors of 8 in order from the smallest to the largest.
2. How many factors does 15 have?
3. List the first five multiples of 3, including the number 3.
4. Listed below are the multiples of 12 and 18.

   12, 24, 36, 48, 60, 72, 84, 96

   18, 36, 54, 72, 90, 102, 114

   What is the LCM?

5. Listed below are the factors of 12 and 18.

   1, 2, 3, 4, 6, 12

   1, 2, 3, 6, 9, 18

   What is the GCF?

IV. Fractions:

A. Converting Fractions to Equivalent Fractions

(1) You can convert one fraction to an equivalent fraction by multiplying the numerator and the denominator by the same number, except for zero.

Examples: (1) $\frac{1}{3} = \frac{1}{3} \times \frac{2}{2} = \frac{1\times 2}{3\times 2} = \frac{2}{6}$

(2) $\frac{4}{7} = \frac{4}{7} \times \frac{3}{3} = \frac{4\times 3}{7\times 3} = \frac{12}{21}$

(3) $\frac{5}{9} = \frac{5}{9} \times \frac{4}{4} = \frac{5\times 4}{9\times 4} = \frac{20}{36}$

(2) Another way to convert from one fraction to an equivalent fraction is to divide the numerator and denominator by a common factor of the numerator and denominator. This is also known as reducing fractions.

Examples: (1) $\frac{16}{24} = \frac{16}{24} \div \frac{2}{2} = \frac{16\div 2}{24\div 2} = \frac{8}{12}$

(2) $\frac{15}{45} = \frac{15}{45} \div \frac{3}{3} = \frac{15\div 3}{45\div 3} = \frac{5}{15}$

(3) $\frac{21}{70} = \frac{21}{70} \div \frac{7}{7} = \frac{21 \div 7}{70 \div 7} = \frac{3}{10}$

B. Reducing Fractions to their Lowest Term

A fraction is in its lowest term when the numerator and denominator can no longer be divided by a common factor but 1. There are two ways to reduce a fraction to its lowest term.

The 1st way: Divide the numerator and denominator in the fraction by any common factor, except 1 and 0. Keep repeating until there are no more common factors.

Examples: (1) $\frac{16}{24} = \frac{16}{24} \div \frac{2}{2} = \frac{16 \div 2}{24 \div 2} = \frac{8}{12} = \frac{8}{12} \div \frac{2}{2} = \frac{8 \div 2}{12 \div 2} = \frac{4}{6} =$
$\frac{4}{6} \div \frac{2}{2} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

(2) $\frac{27}{81} = \frac{27}{81} \div \frac{3}{3} = \frac{27 \div 3}{81 \div 3} = \frac{9}{27} = \frac{9}{27} \div \frac{3}{3} = \frac{9 \div 3}{27 \div 3} = \frac{3}{9} =$
$\frac{3}{9} \div \frac{3}{3} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}$

The 2nd way: Divide the numerator and denominator in the fraction by their Greatest Common Factor (GCF).

Examples: (1) $\frac{16}{24} = \frac{16}{24} \div \frac{8}{8} = \frac{16 \div 8}{24 \div 8} = \frac{2}{3}$

(2) $\frac{27}{81} = \frac{27}{81} \div \frac{27}{27} = \frac{27 \div 27}{81 \div 27} = \frac{1}{3}$

Assignment: Reduce the Fractions to their Lowest Term

1. $\frac{9}{15}$
2. $\frac{21}{49}$
3. $\frac{24}{96}$
4. $\frac{27}{99}$

5. $\frac{42}{48}$
6. $\frac{36}{60}$

C. Changing Mixed Numbers to Improper Fractions

In order to change a mixed number to an improper fraction, you must first multiply the whole number by the denominator, and then add that number to the numerator.

Example: $2\frac{1}{3}$

First take the whole number, 2, and multiply it by the denominator, 3, to get $2 \times 3 = 6$. Then add this number, 6, to the numerator, 1, to get $6 + 1 = 7$. Now the mixed number can be rewritten as an improper fraction. So,

$2\frac{1}{3} = \frac{7}{3}$ or, it can be expressed mathematically

$$2\frac{1}{3} = \frac{((2\times 3)+1)}{3} = \frac{7}{3}$$

Assignment: Changing Mixed Numbers to Improper Fractions

1. $3\frac{1}{4}$
2. $6\frac{7}{8}$
3. $9\frac{5}{7}$
4. $4\frac{2}{3}$
5. $11\frac{5}{12}$

D. Changing Improper Fractions to Mixed Numbers

In order to change an improper fraction to a mixed number, you must first divide the numerator by the denominator. The remainder, *r*, then becomes the new numerator of the fraction.

Examples: (1) $\frac{7}{3} = 7 \div 3 = 2$ with a remainder of $1 = 2\,r1 = 2\frac{1}{3}$

(2) $\dfrac{83}{25} = 83 \div 25 = 3$ with a remainder of $8 = 3\,r8 = 3\dfrac{8}{25}$

Assignment: Change the Improper Fractions to Mixed Numbers

1. $\dfrac{11}{3}$
2. $\dfrac{34}{12}$
3. $\dfrac{76}{9}$
4. $\dfrac{73}{21}$
5. $\dfrac{95}{7}$

E. Adding Fractions

(1) When the fractions have like denominators, there are only two steps to follow. First, you add the numerators and then reduce the fraction to its lowest term.

Examples: (1) $\dfrac{4}{35} + \dfrac{7}{35} = \dfrac{4+7}{35} = \dfrac{11}{35}$

(2) $\dfrac{3}{12} + \dfrac{1}{12} = \dfrac{3+1}{12} = \dfrac{4}{12} = \dfrac{4 \div 4}{12 \div 4} = \dfrac{1}{3}$

(3) $\dfrac{4}{15} + \dfrac{16}{15} = \dfrac{4+16}{15} = \dfrac{20}{15} = \dfrac{20 \div 5}{15 \div 5} = \dfrac{4}{3}$

(2) When the fractions have unlike denominators, there are several steps to follow. First you find the LCD (Least Common Denominator) of the fractions, then convert each denominator to the LCD, add the numerators and reduce the fraction to its lowest term.

Examples: (1) $\dfrac{3}{4} + \dfrac{1}{8}$

(1st find the LCD) LCD = 8

$\dfrac{3 \times 2}{4 \times 2} + \dfrac{1}{8} = \dfrac{6}{8} + \dfrac{1}{8} = \dfrac{6+1}{8} = \dfrac{7}{8}$

(2) $\dfrac{5}{3} + \dfrac{1}{5}$

(1st find the LCD) LCD = 15

$$\frac{5\times5}{3\times5}+\frac{1\times3}{5\times3}=\frac{25}{15}+\frac{3}{15}=\frac{25+3}{15}=\frac{28}{15}$$

(3) When you add mixed numbers, you need to change the mixed numbers to improper fractions then add them as fractions according to the above steps.

Examples: (1) $1\frac{4}{35}+3\frac{7}{35}$

1st convert the mixed numbers to improper fractions:

$1\frac{4}{35}=\frac{39}{35}$ and $3\frac{7}{35}=\frac{112}{35}$

$$1\frac{4}{35}+3\frac{7}{35}=\frac{39}{35}+\frac{112}{35}=\frac{39+112}{35}=\frac{151}{35}$$

converting back to a mixed number you get: $\frac{151}{35}=4\frac{11}{35}$

(2) $2\frac{5}{3}+2\frac{1}{5}$

1st convert the mixed numbers to improper fractions:

$2\frac{5}{3}=\frac{11}{3}$ and $2\frac{1}{5}=\frac{11}{5}$

(1st find the LCD) LCD = 15

$$\frac{11\times5}{3\times5}+\frac{11\times3}{5\times3}=\frac{55}{15}+\frac{33}{15}=\frac{55+33}{15}=\frac{88}{15}=5\frac{13}{15}$$

Assignment: Add the following Fractions (reduce all fractions to lowest terms)

1. $\frac{4}{23}+\frac{7}{23}$
2. $\frac{6}{45}+\frac{9}{45}$
3. $\frac{2}{3}+\frac{5}{9}$
4. $\frac{1}{4}+\frac{3}{7}$
5. $1\frac{4}{23}+3\frac{11}{23}$
6. $2\frac{3}{4}+4\frac{2}{5}$

F. Subtracting Fractions

(1) When the fractions have like denominators, there are only two steps to follow. First, you subtract the numerators and then reduce the fraction to its lowest term.

Examples: (1) $\frac{7}{35}$ - $\frac{4}{35} = \frac{7-4}{35} = \frac{3}{35}$

(2) $\frac{3}{12}$ - $\frac{1}{12} = \frac{3-1}{12} = \frac{2}{12} = \frac{2 \div 2}{12 \div 2} = \frac{1}{6}$

(2) When the fractions have unlike denominators, there are several steps to follow. First, you find the LCD (Least Common Denominator) of the fractions, then convert each denominator to the LCD, subtract the numerators and reduce the fraction to its lowest term.

Examples: (1) $\frac{5}{4}$ - $\frac{2}{3}$

(1st find the LCD) LCD = 12

$\frac{5 \times 3}{4 \times 3}$ - $\frac{2 \times 4}{3 \times 4} = \frac{15}{12}$ - $\frac{8}{12} = \frac{15-8}{12} = \frac{7}{12}$

(2) $\frac{8}{5}$ - $\frac{2}{7}$

(1st find the LCD) LCD = 35

$\frac{8 \times 7}{5 \times 7}$ - $\frac{2 \times 5}{7 \times 5} = \frac{56}{35}$ - $\frac{10}{35} = \frac{56-10}{35} = \frac{46}{35}$

(3) When you subtract mixed numbers, you need to change the mixed numbers to improper fractions then subtract them as fractions according to the above steps.

Examples: (1) $3\frac{7}{35}$ - $1\frac{4}{35}$

1st convert the mixed numbers to improper fractions:

$3\frac{7}{35} = \frac{112}{35}$ and $1\frac{4}{35} = \frac{39}{35}$

$3\frac{7}{35}$ - $1\frac{4}{35} = \frac{112}{35}$ - $\frac{39}{35} = \frac{112-39}{35} = \frac{73}{35}$

converting back to a mixed number you get: $\frac{73}{35} = 2\frac{3}{35}$

(2) $2\frac{5}{3}$ - $2\frac{1}{5}$

1st convert the mixed numbers to improper fractions:

$2\frac{5}{3} = \frac{11}{3}$ and $2\frac{1}{5} = \frac{11}{5}$

(1st find the LCD) LCD = 15

$$\frac{11\times5}{3\times5} - \frac{11\times3}{5\times3} = \frac{55}{15} - \frac{33}{15} = \frac{55-33}{15} = \frac{22}{15} = 1\frac{7}{15}$$

Assignment: Subtract the following Fractions (reduce all fractions to lowest terms)

1. $\frac{7}{23} - \frac{4}{23}$
2. $\frac{9}{45} - \frac{6}{45}$
3. $\frac{2}{3} - \frac{5}{9}$
4. $\frac{3}{7} - \frac{1}{4}$
5. $3\frac{11}{23} - 1\frac{4}{23}$
6. $4\frac{2}{5} - 2\frac{3}{4}$

G. Multiplying Fractions

(1) Multiplying Fractions by Whole Numbers

When you multiply a fraction by a whole number, let the denominator of the whole number be 1 and multiply as fractions.

Examples: (1) $\frac{5}{11} \times 7 = \frac{5}{11} \times \frac{7}{1} = \frac{5\times7}{11\times1} = \frac{35}{11}$

(2) $15 \times \frac{2}{9} = \frac{15}{1} \times \frac{2}{9} = \frac{15\times2}{1\times9} = \frac{30}{9}$ (don't forget to simply!)

$\frac{30}{9} = \frac{30\div3}{9\div3} = \frac{10}{3}$

(2) Multiplying Fractions by Fractions

When multiplying fractions by fractions, simply multiply the numerators together and the denominators together, then simplify.

Examples: (1) $\frac{5}{8} \times \frac{3}{7} = \frac{5\times3}{8\times7} = \frac{15}{56}$

(2) $\frac{4}{9} \times \frac{9}{11}$

When a numerator and denominator are the same number, you can cancel them out!

Since there is a 9 in the numerator and denominator, we can cancel them out. This gives us the following result.

$\frac{4}{9} \times \frac{9}{11} = \frac{4 \times 1}{1 \times 11} = \frac{4}{11}$

(3) $\frac{4}{21} \times \frac{35}{16}$

In this case, we have common factors of 4 in the numerator of 4 and the denominator of 16 and of 7 in the numerator of 35 and the denominator of 21. We can cancel the common factors to give us the following result.

$\frac{4}{21} \times \frac{35}{16} = \frac{1}{3} \times \frac{5}{4} = \frac{1 \times 5}{3 \times 4} = \frac{5}{12}$

(3) Multiplying Mixed Numbers

When multiplying mixed numbers, you need to first convert them to improper fractions and then multiply.

Examples: (1) $2\frac{2}{3} \times 4\frac{1}{5}$

1st convert the mixed numbers to improper fractions

$2\frac{2}{3} = \frac{8}{3}$ and $4\frac{1}{5} = \frac{21}{5}$ So,

$2\frac{2}{3} \times 4\frac{1}{5} = \frac{8}{3} \times \frac{21}{5} = \frac{8 \times 21}{3 \times 5} = \frac{168}{15} = 11\frac{3}{15} = 11\frac{1}{5}$

(2) $3\frac{2}{3} \times \frac{1}{5}$

1st convert the mixed numbers to improper fractions

$3\frac{2}{3} = \frac{11}{3}$ So,

$3\frac{2}{3} \times \frac{1}{5} = \frac{11}{3} \times \frac{1}{5} = \frac{11 \times 1}{3 \times 5} = \frac{11}{15}$

Assignment: Multiply the following Fractions (reduce all fractions to lowest terms)

1. $6 \times \frac{2}{18}$

2. $\frac{6}{7} \times \frac{2}{5}$

3. $\frac{4}{9} \times \frac{3}{8}$

4. $\frac{8}{21} \times \frac{49}{16}$

5. $1\frac{2}{3} \times 2\frac{1}{5}$

6. $4\frac{1}{3} \times 3\frac{2}{5}$

H. Dividing Fractions

(1) Dividing by a Fraction

When you divide a number by a fraction, you just multiply the first number by the reciprocal of the second number.

Examples: (1) $8 \div \frac{1}{3}$

1st get the reciprocal of the second number: reciprocal of $\frac{1}{3}$ is $\frac{3}{1}$

2nd multiply the first number by the reciprocal:

$8 \div \frac{1}{3} = 8 \times \frac{3}{1} = 8 \times 3 = 24$

(2) $\frac{1}{3} \div 8$

1st get the reciprocal of the second number: reciprocal of 8 is $\frac{1}{8}$

2nd multiply the first number by the reciprocal:

$\frac{1}{3} \div 8 = \frac{1}{3} \times \frac{1}{8} = \frac{1\times1}{3\times8} = \frac{1}{24}$

(3) $\frac{1}{3} \div \frac{2}{7}$

1st get the reciprocal of the second number: reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$

2nd multiply the first number by the reciprocal:

$\frac{1}{3} \div \frac{2}{7} = \frac{1}{3} \times \frac{7}{2} = \frac{1\times7}{3\times2} = \frac{7}{6}$ or $1\frac{1}{6}$

(2) Dividing by a Mixed Number

When you divide by mixed numbers, remember to always convert them to improper fractions first. Once you have done that, you simply multiply the first number by the reciprocal of the second number.

Examples: (1) $2\frac{1}{3} \div \frac{9}{5}$

1st change the mixed number to an improper fraction: $2\frac{1}{3} = \frac{7}{3}$

2nd get the reciprocal of the second number: reciprocal of $\frac{9}{5}$ is $\frac{5}{9}$

3rd multiply the first number by the reciprocal:

$$2\frac{1}{3} \div \frac{9}{5} = \frac{7}{3} \div \frac{9}{5} = \frac{7}{3} \times \frac{5}{9} = \frac{7\times5}{3\times9} = \frac{35}{27} = 1\frac{8}{27}$$

(2) $2\frac{1}{3} \div 3\frac{2}{5}$

1st change the mixed numbers to improper fractions:

$2\frac{1}{3} = \frac{7}{3}$ and $3\frac{2}{5} = \frac{17}{5}$

2nd get the reciprocal of the second number: reciprocal of $\frac{17}{5}$ is $\frac{5}{17}$

3rd multiply the first number by the reciprocal:

$$2\frac{1}{3} \div 3\frac{2}{5} = \frac{7}{3} \div \frac{17}{5} = \frac{7}{3} \times \frac{5}{17} = \frac{7\times5}{3\times17} = \frac{35}{51}$$

Assignment: Divide the following Fractions (reduce all fractions to lowest terms)

1. $6 \div \frac{1}{9}$
2. $\frac{1}{9} \div 6$
3. $\frac{2}{3} \div \frac{1}{5}$
4. $1\frac{1}{3} \div \frac{5}{6}$
5. $2 \div 1\frac{5}{7}$
6. $5\frac{1}{4} \div 2\frac{2}{7}$

V. Ratios

Examples: (1) 5 to 7 or 5:7 or $\frac{5}{7}$ or 5/7

The ratio is 5 to 7

(2) Mike has a bag with 4 books, 6 notebooks, 1 video game and 3 CD's.

a. What is the ratio of books to video games?

4:1

b. What is the ratio of CD's to the total number of items in the bag?

3 CD's and 4 + 6 + 1 + 3 = 14 total items so the ratio is 3 to 14

Assignment: Ratio Problems

1. When reading the ratio 8:12, we say eight ____ twelve.

There are 29 students in a sixth grade class. There are 14 girls. Find the following:

2. The ratio of the number of girls to the number of students in the class. ___:___
3. The ratio of the number of boys to the number of students in the class. ___:___
4. The ratio of the number of girls to the number of boys in the class. ___:___

VI. Percent

Examples: (1) 25% = 25 to 100 = $^{25}/_{100}$ = $^{1}/_{4}$ = 0.25

(2) 47% = 47 to 100 = $^{47}/_{100}$ = 0.47

A. How to Write a Percent as a Decimal

To write a percent as a decimal, you need to follow two simple steps. First, take away the percent sign (%) and then move the decimal point two places to the left. Add in zeros to the number as needed.

Examples: (1) Write 32% as a decimal

1st take away the % and get 32

2nd move the decimal point two places to the left to get .32

So, 32% = 0.32

(2) Write 27.5% as a decimal

1st take away the % and get 27.5

2nd move the decimal point two places to the left to get .275

So, 27.5% = 0.275

(3) Write 4% as a decimal
1st take away the % and get 4
2nd move the decimal point two places to the left to get .04
So, 4% = 0.04

B. How to Write a Percent as a Fraction

To write a percent as a fraction, you need to follow two to three simple steps. First, take away the percent sign (%) and then place the number over 100. If the numerator is still a decimal, multiply both the numerator and denominator by the same multiple of 10 (10, 100, 1000, etc.) in order to get the numerator to be a whole number. Don't forget to reduce your fraction when you can!

Examples: (1) Write 31% as a fraction
1st take away the % and get 31
2nd place 31 over 100
So, $31\% = \frac{31}{100}$

(2) Write 27.3% as a fraction
1st take away the % and get 27.3
2nd place 27.3 over 100
So, $27.3\% = \frac{27.3}{100}$
Since the numerator is a decimal, we need to make it a whole number,
3rd make 27.3 a whole number
to do this we must multiply it and the denominator by 10
$$\frac{27.3}{100} \times \frac{10}{10} = \frac{273}{1000}$$
So, $27.3\% = \frac{273}{1000}$

(3) Write 4% as a fraction
1st take away the % and get 4
2nd place 4 over 100
$4\% = \frac{4}{100}$
But we're not finished yet! Remember what I said about reducing the fraction when possible. Since both the numerator and denominator can be divided by the same number, we must reduce the fraction.
$$\frac{4}{100} \div \frac{4}{4} = \frac{1}{25}$$
So, $4\% = \frac{1}{25}$

C. How to Write a Decimal as a Percent

To write a decimal as a decimal, you need to follow two simple steps. First, move the decimal point two places to the right and then place a percent sign (%) behind the number.

Examples: (1) Write 0.32 as a percent

1st move the decimal point two places to the right to get 32
2nd place a percent sign (%) behind the number to get 32%
So, 0.32 = 32%

(2) Write 0.275 as a percent

1st move the decimal point two places to the right to get 27.5
2nd place a percent sign (%) behind the number to get 27.5%
So, 0.275 = 27.5%

(3) Write 0.04 as a percent

1st move the decimal point two places to the right to get 4
2nd place a percent sign (%) behind the number to get 4%
So, 0.04 = 4%

D. How to Write a Fraction as a Percent

There are two ways to do this.

The first way: When the fraction is over 100, the denominator is 100, there are only two steps to follow. First, remove the denominator and then add a percent sign (%).

Example: Write $\frac{31}{100}$ as a percent

1st remove the denominator to get 31
2nd add a percent sign
So, $\frac{31}{100} = 31\%$

The second way: Divide the numerator by the denominator, multiply the answer by 100, and add a percent sign (%). This works for all fractions!

Example: (1) Write $\frac{5}{8}$ as a percent

1st divide the numerator by the denominator

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -\underline{48} \\ 20 \\ -\underline{16} \\ 40 \end{array}$$

$$\begin{array}{r} \underline{-\ 40} \\ 0 \end{array}$$

2nd multiply the answer by 100 to get $0.625 \times 100 = 62.5$

3rd add a percent sign (%) t get 62.5%

So, $\frac{5}{8} = 62.5\%$

Assignment: Percent Problems

Write each percent as a decimal.
1. 3%
2. 77.4%
3. 26%

Write each percent as a fraction.
1. 19%
2. 14.4%
3. 12%

Write each decimal as a percent.
1. 0.36
2. 0.678
3. 0.009

Write each fraction as a percent.
1. $\frac{49}{100}$
2. $\frac{4}{25}$
3. $\frac{7}{8}$

VII. Introduction to Measurement

	Metric System	Customary Measurement System
Capacity	liters, millimeters	cups, pints, quarts, gallons
Length	millimeters, centimeters, meters, kilometers	inches, feet, yards, miles
Weight	grams, kilograms	ounces, pounds, tons
Temperature	degrees Celsius	degrees Fahrenheit

Metric System		Customary Measurement System	
Unit	Abbreviation	Unit	Abbreviation
1 millimeter	1 mm	1 inch	1 in
1 centimeter	1 cm	1 foot	1 ft
1 meter	1 m	1 yard	1 yd
1 kilometer	1 km	1 mile	1 mi
1 liter	1 L	1 cup	1 c
1 millimeter	1 mL	1 pint	1 pt
1 gram	1 g	1 quart	1 qt
1 kilogram	1 kg	1 gallon	1 gal
degrees Celsius	$^{\circ}$C	1 ounce	1 oz
		1 pound	1 lb
		1 ton	1 T
		degrees Fahrenheit	$^{\circ}$F

Assignment: Measurement Problems

Match the measurement unit with the correct measurement system, Customary or Metric.

1. meters
2. ounces
3. yards
4. grams
5. degrees Celsius
6. liter

Match what you are measuring, length, capacity, weight, or temperature when using the given measurements:

7. centimeters
8. Fahrenheit
9. kilograms
10. quarts

VIII. Order of Operations

Remember - first, do the operations in parentheses; next, clear exponents; then, multiply and divide from left to right; and last, add and subtract from left to right.

Examples: (1) $20 \div (4 + 6) \times 3^2 - 6$	Add inside the parentheses
$20 \div 10 \times 3^2 - 6$	Clear the exponent
$20 \div 10 \times 9 - 6$	Multiply and divide
$2 \times 9 - 6$	
$18 - 6$	Add and Subtract
12	

(2) $20 \div 4 + 6 \times 3^2 - 6$ — Clear the exponent

$20 \div 4 + 6 \times 9 - 6$ — Multiply and divide

$5 + 54 - 6$ — Add and Subtract

$59 - 6$

53

Following the rules makes all the difference!

Assignment: Order of Operations Problems

Perform the indicated operations using the rules for order of operations.

1. $2 + 3 \times 4$
2. $2 + 8 - 3$
3. $4/2 + 5$
4. $2 \times (2 + 8)$
5. $2 \times 2 + 8$

Open Response Question on Order of Operations
(KIRIS Grade 8, Mathematics, 1996-97, Common Question A)

George and Shannon solved the following problem:

$7 + 3 \times 4 - 8$

George's response was 32 and Shannon's response was 11.

a. Whose response is correct?
b. Create a new problem using at least three operations and the numbers 2, 4, 6, and 8.
c. Solve the problem you created. Explain how you found the solution.

IX. Integers

A. Absolute Value and Integers

Absolute value of a number *a* is represented by $|a|$. It is always a positive number and represents the distance a number is from zero on the number line.

Examples: (1) $|4| = 4$

(2) $|-4| = 4$

(3) $|0| = 0$

(4) $|250| = 250$

B. Adding Integers

When adding integers, you follow three simple rules:
(1) When you add two positive integers, the result is always a positive integer.
(2) When you add two negative integers, the result is always a negative integer.
(3) When you add a positive and negative integer, you take the absolute value of each integer and subtract those values. The result is this difference and the sign for the result is the sign on the integer with the greatest absolute value.

Examples: (1) 6 + 9 = 15

(2) –7 + –8 = –(7 + 8) = –15

(3) –8 + 15
1st take the absolute value for each integer: $|-8| = 8$ and $|15| = 15$
2nd subtract these values: 15 –8 = 7
3rd use the sign with the large of the absolute values: the sign for 15 is + (positive)
So, –8 + 15 = 7

(4) 8 + –10
1st take the absolute value for each integer: $|8| = 8$ and $|-10| = 10$
2nd subtract these values: 10 – 8 = 2
3rd use the sign with the large of the absolute values: the sign for 10 is – (negative)
So, 8 + –10 = –2

C. Subtracting Integers

When you subtract an integer, all you do is add its opposite. Then, follow the rules for adding integers.

Examples: (1) 6 – 3 = 6 + (–2) = 4

(2) 14 – (–4) = 14 + (4) = 18

(3) –8 – 8 = –8 + (–8) = –16

(4) –16 – (–48) = –16 + (48) = 32

D. Multiplying Integers

When multiplying integers, just follow the simple rules below.

Rules for Multiplying Integers

Sign of Integers		Sign of Product
+ × +	=	+
- × -	=	+
+ × -	=	-
- × +	=	-

Examples: (1) $3 \times 8 = 24$

(2) $-3 \times -8 = 24$

(3) $-9 \times 2 = -18$

(4) $8 \times -1 = -8$

E. Dividing Integers

When dividing integers, just follow the simple rules below.

Rules for Dividing Integers

Sign of Integers		Sign of Product
+ ÷ +	=	+
- ÷ -	=	+
+ ÷ -	=	-
- ÷ +	=	-

Examples: (1) $8 \div 2 = 4$

(2) $-14 \div -7 = 2$

(3) $-10 \div 5 = -2$

(4) $-9 \div -3 = 3$

Assignment: Perform the indicated operation on each of the following Integers

1. $|-24|$
2. $|0|$
3. $|174|$
4. $|-1,259|$
5. $|40,628|$
6. $-5 + -3$
7. $6 + -3$
8. $-6 + -2$
9. $-4 + -6$

10. –3 + 3
11. –6 + 10
12. –4 +6
13. 8 + –8
14. –3 + –8
15. 7 + 9
16. 4 – (–4)
17. 3 – 8
18. 5 – (–6)
19. –10 – 6
20. –10 – (–6)
21. –1 – 8
22. –1 – (–8)
23. –3 – (–3)
24. –6 – 9
25. –5 – (–7)
26. 2 × –6
27. 4 × 3
28. –3 × –5
29. –2 × 5
30. 4 × –1
31. 12 ÷ 2
32. –9 ÷ 3
33. 6 ÷ –3
34. –8 ÷ –2
35. 6 ÷ –2

X. Rational, Irrational and Real Numbers

Review the definitions for irrational, rational and real numbers then complete the following assignment.

Assignment: Rational, Irrational and Real Numbers Assignment

For 1 – 7, state what each number is (irrational, rational and/or real). Some may be more than one type.

1. 2
2. 0.65
3. $\frac{4}{3}$
4. 3.14159
5. $\frac{-12}{4}$
6. $1\frac{1}{3}$

7. 3.1415926535897932……..
8. Are all integers rational numbers?
9. Are all rational numbers integers?
10. Can a number be both rational and irrational?
11. Are all rational numbers real numbers?
12. Are all irrational numbers real numbers?

Answers for Practice Activities in Middle School Math Key Concepts:

I.B.
Adding and Subtracting the following Decimals

1. 7.3
2. 15.7
3. 0.279
4. 8.76
5. 8.564
6. 1.2
7. 0.056
8. 3.5
9. 0.7
10. 2.22

I.D.
Multiplying and Dividing the following Decimals

1. 1.12
2. 20
3. 0.0015
4. 17.36
5. 7.62
6. 1.4
7. 0.091
8. 3
9. 1600
10. 0.27

II.
Prime and Composite Number Chart

Prime Numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Composite Numbers are: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66, 68, 69, 70, 72, 74, 75, 76, 77, 78, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100

II.
Prime and Composite Numbers

1. composite
2. prime
3. composite
4. composite
5. 5×5
6. $2 \times 2 \times 2 \times 3$
7. prime
8. $2 \times 3 \times 5$

III.
GCL and LCM

1. 1, 2, 4, 8
2. 4
3. 3, 6, 9, 12, 15
4. 36
5. 6

IV.B.
Reduce the Fractions to their Lowest Term

1. $\frac{3}{5}$
2. $\frac{3}{7}$
3. $\frac{1}{4}$
4. $\frac{3}{11}$
5. $\frac{7}{8}$
6. $\frac{3}{5}$

IV.C.
Changing Mixed Numbers to Improper Fractions

1. $\frac{13}{4}$
2. $\frac{55}{8}$
3. $\frac{68}{7}$
4. $\frac{14}{3}$
5. $\frac{137}{12}$

IV. D.
Change the Improper Fractions to Mixed Numbers

1. $3\frac{2}{3}$
2. $2\frac{10}{12}$ or $2\frac{5}{6}$
3. $8\frac{4}{9}$
4. $3\frac{10}{21}$
5. $13\frac{4}{7}$

IV.E.
Add the following Fractions

1. $\frac{11}{23}$
2. $\frac{1}{3}$
3. $\frac{11}{9}$ or $1\frac{2}{9}$
4. $\frac{19}{28}$
5. $4\frac{15}{23}$
6. $7\frac{3}{20}$

IV. F.
Subtract the following Fractions

1. $\frac{3}{24}$
2. $\frac{2}{15}$
3. $\frac{1}{9}$
4. $\frac{5}{28}$
5. $2\frac{7}{23}$
6. $1\frac{13}{20}$

IV. G.
Multiply the following Fractions

1. $\frac{2}{3}$
2. $\frac{12}{35}$
3. $\frac{1}{6}$
4. $\frac{7}{6}$ or $1\frac{1}{6}$
5. $\frac{11}{3}$ or $3\frac{2}{3}$
6. $\frac{221}{15}$ or $14\frac{11}{15}$

IV. H.
Divide the following Fractions

1. 54
2. $\frac{1}{54}$
3. $\frac{10}{3}$ or $3\frac{1}{3}$
4. 2

5. $\frac{7}{6}$ or $1\frac{1}{6}$
6. 12

V.
Ratio Problems

1. to
2. 14:29
3. 15:29
4. 14:15

VI.
Percent Problems

Write each percent as a decimal.
1. 0.03
2. 0.774
3. 0.26
Write each percent as a fraction.
1. m
2. $\frac{144}{1000}$
3. $\frac{3}{25}$
Write each decimal as a percent.
1. 36%
2. 67.8%
3. 0.9%
Write each fraction as a percent.
1. 49%
2. 16%
3. 87.5%

VII.
Measurement Problems

1. Metric
2. Customary
3. Customary
4. Metric
5. Metric
6. Metric
7. Length
8. Temperature

9. Weight
10. Capacity

VIII.
Order of Operations Problems

1. 14
2. 7
3. 7
4. 20
5. 12

Open Response

Part a: According to the rules for order of operations, the first operation you would perform in this problem is to multiply 3 × 4. Therefore, 7 + 3 × 4 - 8 becomes 7 + 12 - 8. Then, you perform addition and subtraction as they come from left to right. Therefore, 7 + 12 is 19, then subtracting 8 leaves 11. Shannon is correct.

Part b and c: There are many correct responses for this question so you will need to show your response to your teacher.

IX.
Perform the indicated operation on each of the following Integers

1. 24
2. 6
3. 174
4. 1,259
5. 40,628
6. –8
7. 3
8. –8
9. –10
10. 0
11. 4
12. 2
13. 0
14. –11
15. 16
16. 8
17. –5
18. 11
19. –16
20. –4
21. –9

22. 7
23. 0
24. –15
25. 2
26. –12
27. 12
28. 15
29. –10
30. –4
31. 6
32. –3
33. –2
34. 4
35. -3

X.
Rational, Irrational and Real Numbers

1. rational, real
2. rational, real
3. rational, real
4. rational, real
5. rational, real
6. rational, real
7. irrational, real
8. yes
9. no
10. no
11. yes
12. yes